MEASURING POVERTY USING PARAMETERIZED LORENZ CURVE – THE CASE OF MACEDONIA

MERENJE SIROMAŠTVA UPOTREBOM PARAMETRIZOVANE LORENCOVE KRIVE - SLUČAJ MEKEDONIJE

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Abstract: *Primary aim of this paper is to show how do we construct poverty measures from grouped data, i.e. to show how do we derive poverty measures from parameterized Lorenz curve. In this paper Gaurav Datt's approach has been used. The derived poverty measures are estimated in the case of Macedonia, using interactive software package "Povcal", created by the World Bank.*
In the case of Macedonia, our findings suggest two main conclusions: 1) The high poverty rate is accompanied with the moderate level of income inequality, and 2) The transmission mechanism that leads from economic growth to poverty reduction, is working properly.

Key words: *Parameterized Lorenz curve, General Quadratic Lorenz curve, Foster - Greer - Thorbecke index.*

Sadržaj: *Primarni cilj ovog rada je da pokaže kako možemo konstruisati merila siromaštva iz grupne baze podataka, odnosno da pokaže na koji način merila siromaštva proizilaze iz parametrizovane Lorencove krive. U radu je upotrebljen pristup Gaurav Data. Izvedena merila siromaštva su ocenjena u slučaju Makedonije, koristeći interaktivni softverski paket "Povcal", koji je kreiran sa strane Svetske banke.*
U slučaju Makedonije, rezultati do kojih smo došli sugeriraju dva primarna zaključka: 1) Visoka stopa siromaštva praćena je umerenim nivoom dohodovne nejednakosti, i 2) Mehanizam transmisije koji vodi od ekonomskog rasta do smanjenja siromaštva, radi korektno.

Ključne reči: *parametrizovana Lorencova kriva, opšta kvadratna Lorencova kriva, Foster-Grir-Torbeke indeks.*

1. INTRODUCTION

Grouped data are the most common form of information available to researchers, when it comes to the problem of poverty and income distribution.

There are two basic approaches when constructing poverty measures from grouped data: The interpolation methods [1], and the methods based on parameterized Lorenz curve. In this paper, we use the Gaurav Datt's approach based on a parameterized Lorenz curve [2], with a General Quadratic functional form.

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In accordance with our knowledge, this is the first attempt in Macedonia, on a basis of a parameterized Lorenz curve and grouped data: 1) to construct poverty measures using, so called, P-alfa class of measures, and 2) to calculate their elasticises with respect to the mean income and the Gini index.

The paper structure is as follows: In the first part it is presented review of papers and studies related with the specific area. The second part explains methodological background and sources of data. The third part contains the results from the research, while in the fourth part are being sublimated the concluding remarks.

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2. LITERATURE REVIEW

In the literature not to many often attempts are being made to test the theoretical validity and empirical performances of the alternative functional forms of the Lorenz curve [6]. First of all, Kakwani [3] set the mathematical formulation necessary for parameterization of Beta Lorenz curve, and later Villasenor and Arnold [4] did the same for the General Quadratic (GQ) Lorenz curve.

In his seminal paper Datt [2] showed how we can construct poverty measures when grouped data is available, using Foster-Greer-Torbeke (FGT) class of poverty measures. In the same paper, he explained the ways of constructing point estimates of the poverty measures' elasticities with respect to the mean income and the Gini index. For estimation of the Lorenz curve, Datt uses mathematical formulation i.e. functional forms of the GQ and Beta Lorenz curve, proposed by Kakwani and Villasenor and Arnold.

Essama-Nssah [5], following the procedure proposed by Datt, uses regression analysis to fit the data to a model such as the General Quadratic model. In fact, Essama-Nssah's simulation strategy is a modification of Datt's approach. For a parameterization of the Lorenz curve, he computes the associated first and second order derivatives. Then, he combines these results with an estimate of the mean of the distribution to recover levels of the welfare indicator (using the first order derivative) along with an estimate of the density function (based on the second order derivative).

Minoiu and Reddy [6] asses the performance of functional forms proposed by Kakwani and Villasenor and Arnold to estimated the Lorenz curve from grouped data. The methods are

implemented using the computational tools such as Povcal and SimSIP, both developed by the World Bank. To identify biases associated with these methods, they use unit data from several household surveys and theoretical distributions. They are concluding that poverty and inequality are better estimated when the true distribution is unimodal than multimodal.

More comprehensive poverty and inequality studies, based on parameterized Lorenz curve, for a different functional forms, can be find in: Bhalla [7]; Chen and Ravallion [8]; Figini and Santarelli, [9]; Pritchett [10]; Son and Kakwani [11]; Kamin [12]; Edward and Sumner [13]; Kakwani and Podder [14], [15].

3. METHODOLOGY AND DATA

The methodology is based on the following two functions [2]:

1) Lorenz curve:

$$L = L(p;\pi) \qquad (1)$$

where: L is the share of the aggregate income that belongs to the poorest p percentages of the households, and π is a vector of the Lorenz curve (estimable) parameters.

2) Poverty measure:

$$P = P(\mu / z;\pi) \qquad (2)$$

where: P is a poverty measure given as a function of the coefficient of the mean income μ and the poverty line z, and the parameters of the Lorenz curve π.

The function L covers relative inequalities in the households and supports alternative parameterizations of the Lorenz curve, while the function P, which is homogenous of degree zero in mean income and poverty line[3], covers the assessment of the absolute living standard of the poor households, and supports different poverty measures [2].

Regarding the poverty measures, we use FGT index:

$$P_\alpha = \int_o^z \left[\frac{z-x}{z}\right]^\alpha f(x)dx \quad \alpha \geq 0 \qquad (3)$$

where: x is the household income; $f(x)$ is its density (roughly estimated proportion of households with income x); z is poverty line, and α is nonnegative parameter[4].
FGT index is preferred for this type of research since it is additively separable index[5] and incorporates: Head count index *(H)*; Poverty gap index *(PG)*; and Poverty severity index *(PS)*, where: H corresponds to $\alpha = 0$, PG to $\alpha = 1$, while PS corresponds to $\alpha = 2$.

Given the best performances, usually, the estimation of the Lorenz curve is based on the following two functional forms: GQ Lorenz curve [4] and Beta Lorenz curve [3].

In this paper our focus is aimed at the GQ Lorenz curve, which specification, as well as the derived equations necessary for estimation of poverty measures (H, PG, PS), are given in Annex 1, Table A1.1.

In order to estimate the poverty measures, first we need to estimate the parameters of the GQ Lorenz curve, using the following regression:

$$L(1-L) = a(p^2 - L) + bL(p-1) + c(p-L) \quad (4)$$

The regression (4) does not contains an intercept. The parameters are estimated with the OLS method, using all except the last observation for (*p, L)*. The last observation that takes values (1, 1) is excluded since the functional form for the Lorenz curve already is being established to pass through the points (1, 1).

Afterwards, in order to estimate the poverty measures (see, equations presented in Annex 1, Table A1.1), we need to set out the mean income μ and the poverty line z.

Finally, we have to check whether the parameterization enables theoretically valid Lorenz curve (for the conditions of theoretical validity of the Lorenz curve, see: Annex 1, Table A1.2).

The monthly grouped data has been obtained from the study "Material deprivation poverty and social exclusion in Republic of Macedonia" [16] (see: Annex 2, Table A2.1).

4. RESULTS

The estimated parameters a*, b* and c of the GQ Lorenz curve are presented in Annex 2, Table A2.2. Our Lorenz curve satisfies previously outlined theoretical criteria regarding its validity.

The Gini index counts 37.84 and shows moderate, to high, level of inequality in distribution of incomes between households in the Republic of Macedonia (see: Annex 2, Table A2.3 and Figure A2.1).

The mean income μ is set to a 19073,70 denars (about 347,00 US$), while the poverty line z is set to a 60% of the mean income, or 11444,00 denars (about 208,00 US$), (see: Annex 2, Table A2.3).

The estimation of the Head Count Index *(H)* proves that 33.38% of the total number of households in Macedonia are living below the poverty line, set to a 60% of the household's mean income (see: Annex 2, Table A2.3).

The estimation of the Poverty gap index *(PG)* counts 11.40% (see: Annex 2, Table A2.3) and shows that, monthly, on average, it takes 1304,16 denars (or about 25US$) per household, for poor households to get out of the poverty zone. It means that it takes approximately 217.664.301,00 denars (or about 3.957.532,00 US$) per month, for poor households to pass the poverty line. The Poverty severity index counts 8.49%.[6]

The elasticities of poverty measures with respect to the households' mean income indicates that increase of the mean income for 1%, leads to decrease of the Head count index for about -1.23%, i.e. decrease of the Poverty gap index for -1.32%, which confirms a high level of responsiveness of poverty indices (see: Annex 2, Table A2.4).

The elasticities of poverty measures with respect to the Gini index, show that increase of the Gini index for 1%, leads to increase of the Head count index for about 0.82%, i.e. increase of the Poverty gap index for 2.54%, which corresponds to the level of high responsiveness of the poverty indices (see: Annex 2, Table A2.4).

5. CONCLUSION

The estimation of the Lorenz curve parameters, which functional form is given as a General Quadratic, confirmed that the Gini index counts 37.84, and indicates moderate, to high level, inequality of income distribution between households in Macedonia.

Poverty measures are estimated on a basis of the household's mean income (which is set up on 19.073,70 denars, or about 347,00 US$) and the poverty line (which is set up on 11.444,00 denars, or about 208,00 US$). The Head Count Index counts 33.38%, Poverty gap index - 11.40%, while Poverty severity index - 8.49%.

Therefore, it can be withdrawn one particularly interesting conclusion about the relationship between the Macedonian poverty rate and inequality index. Namely, the high level of poverty, accompanied with the moderate level of inequality and low households' mean income, suggests that the only thing that is moderately unequally distributed between households in Macedonia, actually is the poverty.

The elaticitices of poverty measures, show high level of responsiveness of the poverty indices in respect with the mean income of households. The same conclusion can be withdrawn for the elasticitices of poverty measures in respect with the Gini index.

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This finding is of a special importance for the creators of economic and social policy in Macedonia, since it leads to the conclusion that the transmission mechanism, from the economic growth towards to poverty reduction - works properly. Furthermore, it confirms that, in the case of Macedonia, there are no signs for, so called, ruthless growth.

Further analysis of the poverty determinants, as well as of the necessary socio-economic policies for poverty alleviation, requires decomposition of changes in poverty rate into growth and redistribution components [17], [18].

NOTES

[3] If the poverty line and mean income change in same proportion, poverty will remain unchanged.
[4] Higher value of the parameter α means higher sensitivity of the measure with respect to the inequality of the poor households.
[5] FGT belongs to the class of additively separable poverty indices, which means that deprivation that one household feels depends only on a fixed poverty line and its level of welfare, but not on the welfare of other households. So, if z is the poverty line, n is the number of households, x_i is the level of welfare of the household i, and $\psi(z, x_i)$ is the indicator of deprivation at the household's level, than this class poverty measures give the average deprivation of the total number of households: $P(z,x) = \frac{1}{n}\sum_{i=1}^{n}\psi(z,x_i)$. When the households are divided into a groups, this class of measures allows one to compute the overall poverty as a weighted average of poverty in each group. The weights here are equal to households' shares. Thus such indices are also additively decomposable [5].
[6] This index is useful for intertemporal comparison of the severity of poverty in certain country, or for country ranking.

ANNEX 1.

	GQ Lorenz curve
Lorenz curve equation $L(p)$	$L(1-L) = a(p^2 - L) + bL(p-1) + c(p-L)$ or $L(p) = -\frac{1}{2}\left[bp + e + (mp^2 + np + e^2)^{1/2}\right]$
(H)	$H = -\frac{1}{2m}\left[n + r(b + \frac{2z}{\mu}\{(b + 2z/\mu)^2 - m\}^{-\frac{1}{2}}\right]$
(PG)	$PG = H - (\mu/z)L(H)$
(PS)	$PS = 2(PG) - H$ $-\left(\frac{\mu}{z}\right)^2\left[aH + bL(H) - \left(\frac{r}{16}\right)\ln\left(\frac{1-H/s_1}{1-H/s_2}\right)\right]$
Note:	$e = -(a+b+c+1)$ $m = b^2 - 4a$ $n = 2be - 4c$ $r = (n^2 - 4me^2)^{1/2}$ $s_1 = (r-n)/(2m)$ $s_2 = -(r+n)/(2m)$

Table A1.1: Poverty measures derived from the parameterized *GQ* Lorenz curve
Source: Datt, G.(1998).

Theoretically valid Lorenz curve satisfies following four conditions:

1) $L(0;\pi)=0$; 2) $L(1;\pi)=1$; 3) $L'(0^+;\pi)\geq 0$; 4) $L''(p;\pi)\geq 0$ za $p\in(0,1)$

The first two conditions are about that 0 and 100 percent of the households gets 0 and 100 percent of the total income, respectively. The third and fourth condition means that Lorenz curve is monotonically increasing and convex.

The equations for the first and second derivative of the GQ Lorenz curve, as well as the conditions for the GQ Lorenz curve validity, are presented below:

$$L'(p)=-\frac{b}{2}-\frac{(2mp+n)(mp^2+np+e^2)^{-1/2}}{4}; \qquad L''(p)=\frac{r^2(mp^2+np+e^2)^{-3/2}}{8}$$

Condition	GQ Lorenz curve
first	$e<0$
second	$a+c\geq 1$
third	$c\geq 0$
fourth	(1) $m<0$ or (2) $0<m<(n^2/(4e^2)),\quad n\geq 0$ or (3) $0<m<-(n/2),\quad m<(n^2/(4e^2))$

Table A1.2: Conditions for theoretical validity of the Lorenz curve

ANNEX 2.

Monthly income per household, in denars	*P*	*L*
0-3000	0.0730	0.00918
3001-6000	0.1320	0.02310
6001-9000	0.2470	0.06832
9001-12000	0.3610	0.13108
12001-15000	0.4730	0.21035
15001-18000	0.5430	0.27090
18001-21000	0.6400	0.37007
21001-24000	0.7110	0.45382
24001-27000	0.7580	0.51666
27001-30000	0.8520	0.65711
30001-45000	0.9340	0.81833
45001 and above	1.0000	1.00000

Table A2.1: Distribution of monthly incomes of households in Macedonia, 2012
note: *p* = cumulative proportion (or percentage) from total number of households;
L = cumulative proportion (or percentage) of monthly income

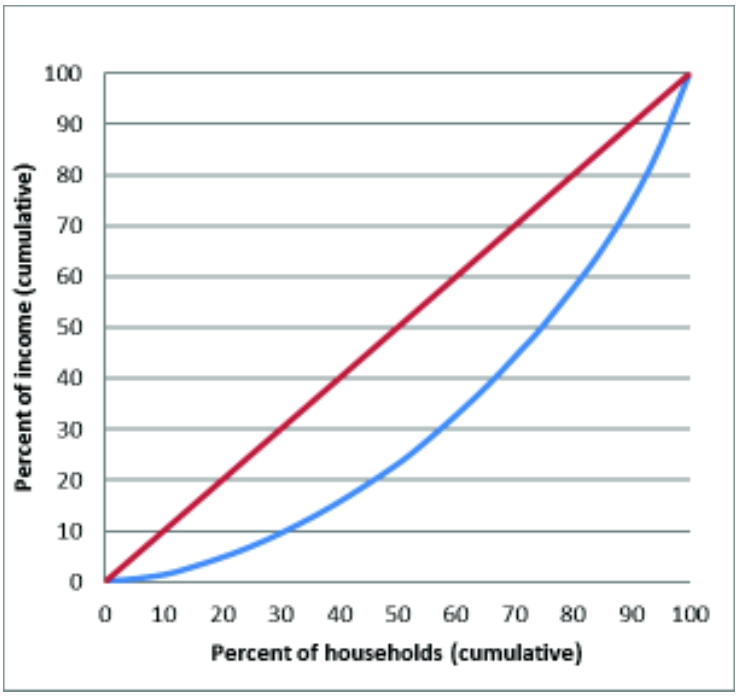

Figure A2.1: GQ Lorenz curve – Macedonia, 2012

Dependent Variable: L*(1-L)
Method: Least Squares
Sample: 1 12
Included observations: 12
L*(1-L)=a*(P^2-L)+b*(L*(P-1))+c*(P-L)

	Coefficient	Std. Error	t-Statistic	Prob.
a	1.430599	0.025493	56.11786	0.0000
b	-1.194924	0.038519	-31.02190	0.0000
c	0.062716	0.014898	4.209749	0.0023

R-squared	0.999814	Mean dependent var	0.139793
Adjusted R-squared	0.999772	S.D. dependent var	0.095691
S.E. of regression	0.001443	Akaike info criterion	-10.03128
Sum squared resid	1.88E-05	Schwarz criterion	-9.910053
Log likelihood	63.18768	Hannan-Quinn criter.	-10.07616
Durbin-Watson stat	1.754458		

Table A2.2: Regression results – *GQ* Lorenz curve

	In denars
Poverty line	11444,00
Mean income	19073,70
	%
Head Count index (H)	33.38

Poverty Gap index (PG)	11.40
Poverty Severity index (PS)	8.49
Gini index	37.84

Table A2.3: Poverty line, mean income, poverty measures and Gini index for Macedonia, 2012

Poverty measures	Mean income	Gini index
Head Count Index *(H)*	-1.23009	0.82010
Poverty Gap Index *(PG)*	-1.31852	2.54575
Poverty Severity Index	-1.39075	4.26061

Table A2.4: Elasticitices of the poverty measures in respect with the mean income and Gini index

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